

7 More Graphing Advanced Rational Functions

Warmup - find the vertical asymptotes, if any.

1 of 10

1. $f(x) = \frac{x}{x+4}$ $x = -4$

2. $f(x) = \frac{x}{x-3}$ $x = 3$

3. $f(x) = \frac{x+3}{x(x+4)}$ $x = 0, -4$

4. $g(x) = \frac{x+3}{x(x-3)}$ $x = 0, 3$

5. $h(x) = \frac{x}{x(x+4)}$ $x = -4$
 $x = 0$ hole

6. $f(x) = \frac{x}{x(x-3)}$ $x = 3$
 $x = 0$ hole

Find the horizontal asymptotes, if any.

7. $f(x) = \frac{12x}{3x^2+1}$ $y = 0$

6. $f(x) = \frac{15x}{3x+1}$ $y = 5$

7 More Graphing Advanced Rational Functions

Asymptotes and Intercepts

2 of 10

Find the asymptotes, intercepts, and holes of the function.
Sketch the graph.

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 - 4}$$

Vertical asymptotes

$$x = 2$$

Horizontal asymptotes

$$y = 2$$

x-intercepts

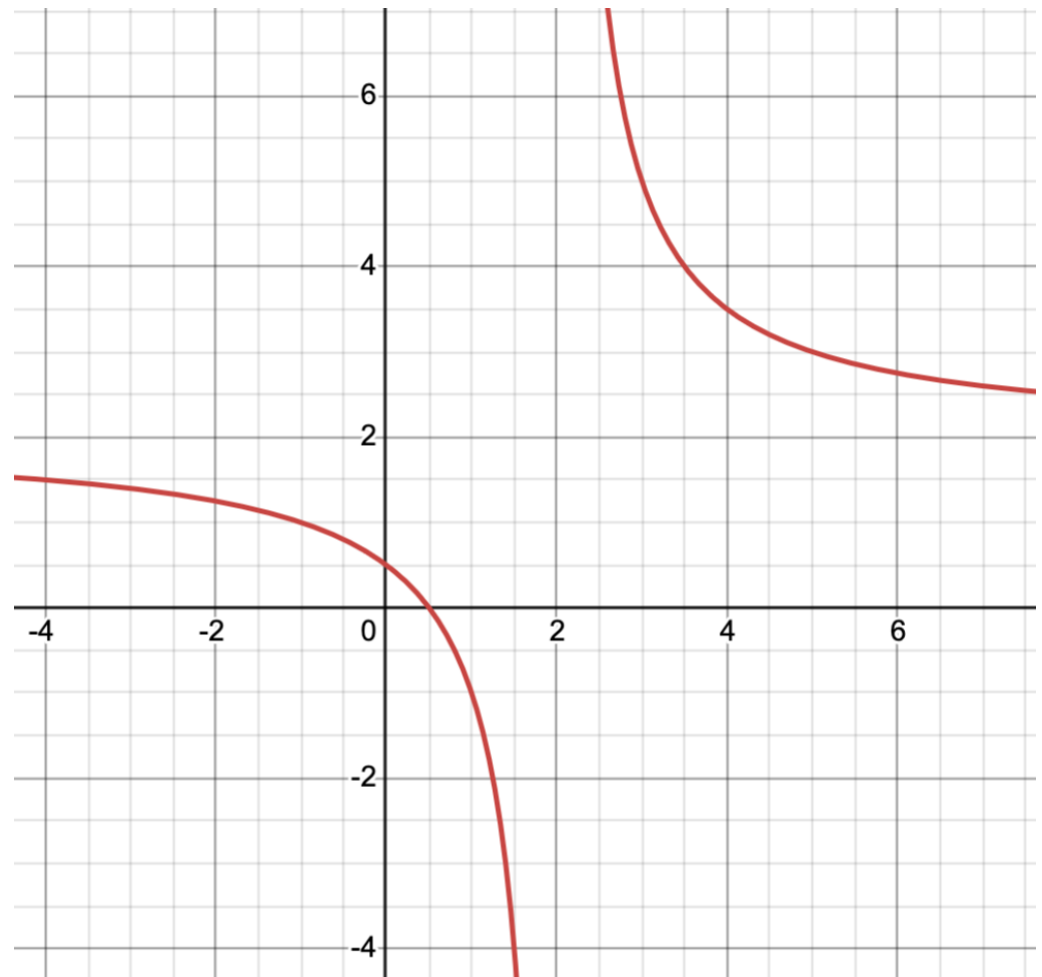
$$x = \frac{1}{2}$$

y-intercepts

$$y = \frac{1}{2}$$

Holes

$$x = -2$$



7 More Graphing Advanced Rational Functions

Graphing

$$f(x) = \frac{-(x+3)}{(x+1)^2(x^2+x-2)}$$

3 of 10

Vertical asymptotes

$$x = -1, -2, 1$$

x-intercepts

$$x = -3$$

y-intercepts

$$y = \frac{3}{2}$$

Horizontal asymptotes

$$y = 0$$

End behavior

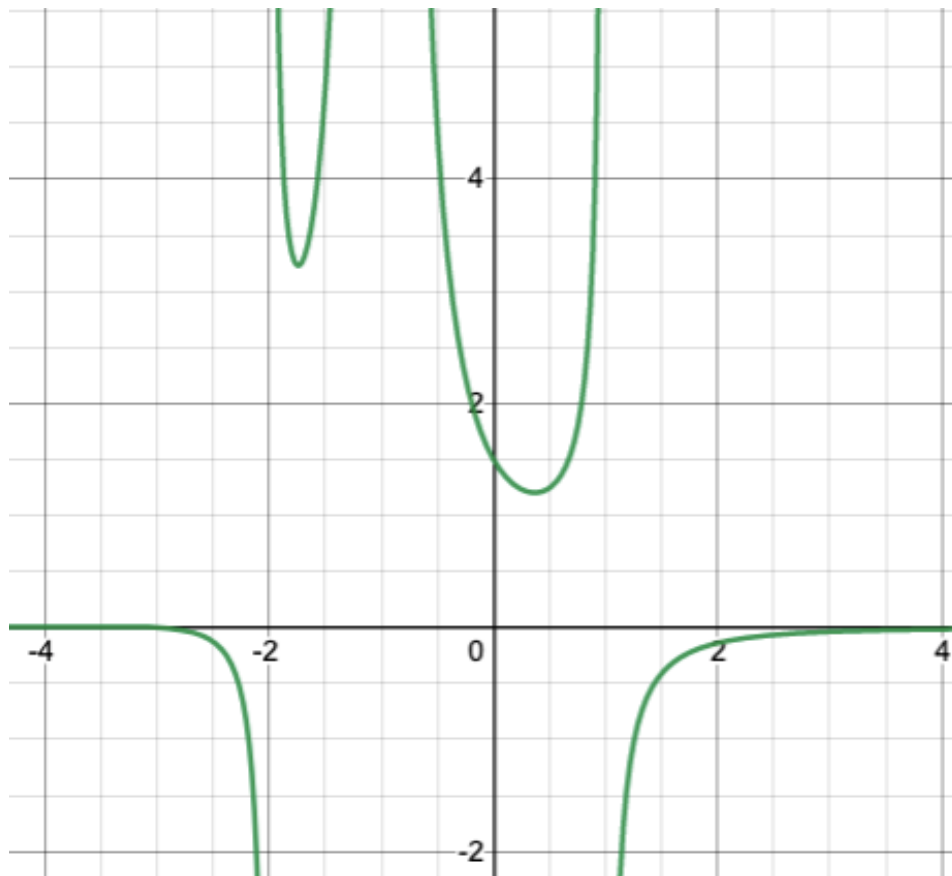
Left/right approach $y=0$

Behavior near vertical asymptotes

$$1^+ = -, 1^- = +$$

$$-1^+ = +, -1^- = +$$

$$-2^+ = +, -2^- = -$$



7 More Graphing Advanced Rational Functions

Finding Asymptotes

4 of 10

Vertical asymptotes occur at the zeros of the denominator provided that the zeros are not also zeros of the numerator of equal or greater multiplicity.

X-intercepts occur at the zeros of the numerator which are not also zeros of the denominator.

Y-intercept is the value of $f(0)$ if defined.

$$f(x) = \frac{2x + 1}{x + 3}$$

Asymptotes: $x = -3$

x-intercepts: $x = -\frac{1}{2}$

y-intercept: $y = \frac{1}{3}$

$$f(x) = \frac{2x + 1}{(2x + 1)(x - 3)}$$

$x = 3$

Hole at $x = -\frac{1}{2}$

none

$y = -\frac{1}{3}$

$$f(x) = \frac{x + 1}{(x + 1)^2}$$

$x = -1$

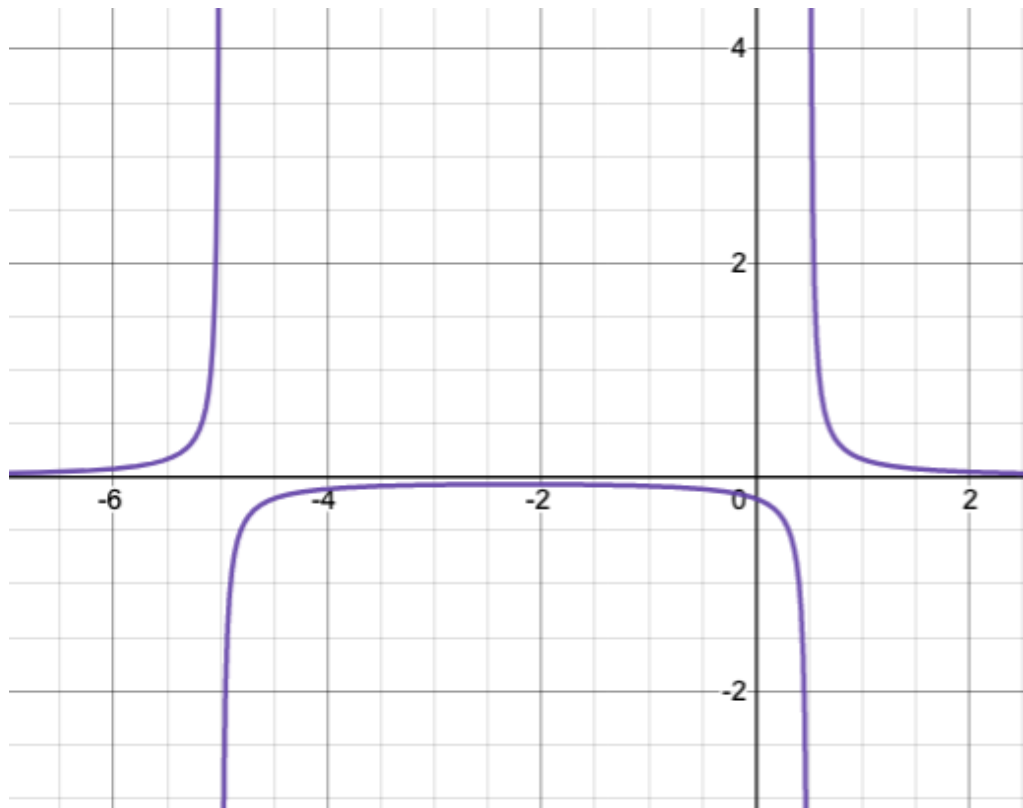
none

$y = 1$

7 More Graphing Advanced Rational Functions

Practice

$$f(x) = \frac{1}{2x^2 + 9x - 5}$$



Vertical asymptotes

$$x = \frac{1}{2}, -5$$

x-intercepts

none

y-intercepts

$$y = -\frac{1}{5}$$

Horizontal asymptotes

$$y = 0$$

End behavior

Left/right approach $y=0$

Behavior near vertical asymptotes

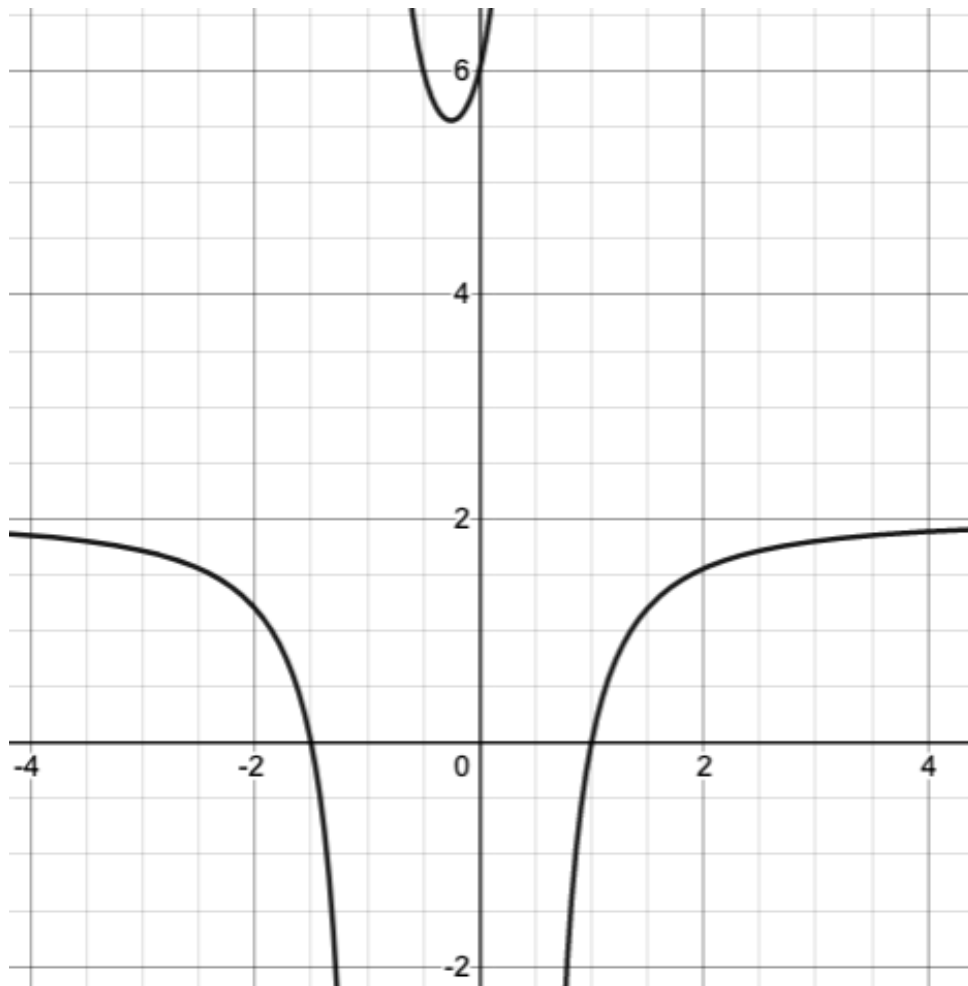
$$\frac{1}{2}^+ = \infty, \frac{1}{2}^- = -\infty$$

$$-5^+ = -\infty, -5^- = \infty$$

7 More Graphing Advanced Rational Functions

Practice

$$f(x) = \frac{-4}{2x^2 + x - 1} + 2$$



Vertical asymptotes

$$x = \frac{1}{2}, -1$$

x-intercepts

$$x = -\frac{3}{2}, 1$$

y-intercepts

$$y = 6$$

Horizontal asymptotes

$$y = 2$$

End behavior

Left/right approach $y=2$

Behavior near vertical asymptotes

$$\frac{1}{2}^+ = -\infty, \frac{1}{2}^- = \infty$$

$$-1^+ = \infty, -1^- = -\infty$$



More Solving Rational Equations

7 More Solving Rational Equations

7 of 10

$$\frac{3}{x^2 - 7x + 10} + 2 = \frac{x - 4}{x - 5}$$

$$\frac{3}{(x - 2)(x - 5)} + \frac{2 \cdot (x - 2)(x - 5)}{(x - 2)(x - 5)} = \frac{(x - 4) \cdot (x - 2)}{(x - 5) \cdot (x - 2)}$$

$x \neq 2, 5$

$$3 + 2 \cdot (x^2 - 7x + 10) = x^2 - 6x + 8$$

$$x^2 - 8x + 15 = 0$$

$$3 + 2x^2 - 14x + 20 = x^2 - 6x + 8$$

$$(x - 3)(x - 5) = 0$$

$x = 3, 5$

Practice

$$\frac{2x}{x - 1} + \frac{1}{x - 3} = \frac{2}{x^2 - 4x + 3}$$

$$x = -\frac{1}{2}$$

3 extraneous

7 More Solving Rational Equations

8 of 10

$$\frac{3}{x^2 - 7x + 10} + 2 = \frac{x - 4}{x - 5}$$

$$\frac{3}{(x - 2)(x - 5)} + \frac{2 \cdot (x - 2)(x - 5)}{(x - 2)(x - 5)} = \frac{(x - 4) \cdot (x - 2)}{(x - 5) \cdot (x - 2)}$$

$x \neq 2, 5$

$$3 + 2 \cdot (x^2 - 7x + 10) = x^2 - 6x + 8$$

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$$(x - 3)(x - 5) = 0$$

$x = 3, 5$

Practice

$$\frac{5}{x^2 + 4x + 4} = -4 \left(\frac{x - 3}{x^2 - x - 6} \right) + 1$$

$x = -3$

7 More Solving Rational Equations

9 of 10

$$\frac{3x}{4} + 1 > \frac{x-1}{2}$$

$$\frac{3x}{4} + \frac{4}{4} > \frac{2(x-1)}{4}$$

$$3x + 4 > 2x - 2$$

$$x + 4 > -2$$

$$x > -6$$

$$\frac{(x+1)(x-2)}{(x+3)(x-1)} \geq 0 ?$$

$$x < -3 \text{ or } -1 \leq x < 1 \text{ or } x \geq 2$$

7 More Solving Rational Equations

$$\frac{3x}{4} + 1 > \frac{x-1}{2}$$

$$3x + 4 > 2x - 2$$

10 of 10

$$\frac{3x}{4} + \frac{4}{4} > \frac{2(x-1)}{4}$$

$$x + 4 > -2$$

$$x > -6$$

Practice

$$\frac{(x+2)(x-3)^2}{(x+4)(x-2)(x^2+5)} > 0$$

$$-4 < x < -2 \text{ or } 2 < x < 3 \text{ or } x > 3$$

hyperlink

